FOCAL SERIES IN FINITE GROUPS

BY

DONALD GORDON HIGMAN

B.A., University of British Columbia, 1949

A.M., University of Illinois, 1951

AN ABSTRACT OF A THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN
MATHEMATICS IN THE GRADUATE COLLEGE
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ACKNOWLEDGMENT

The author wishes to express his heartfelt gratitude to Professor Reinhold Baer, under whose direction this thesis was prepared.

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If there is given a subgroup S of a [finite] group G, we may ask what information is to be obtained about the structure of G from a knowledge of the location of S in G. Thus, for example, famed theorems of Frobenius and Burnside give criteria for the existence of a normal subgroup N of G such that $G = NS$ and $1 = N \wedge S$ — and hence in particular for the non-simplicity of G. To aid in locating S in G, and to facilitate exploitation of the transfer, we single out a descending chain of normal subgroups of S. Namely, we introduce the *focal series* of S in G by means of the recursive formulas

$S(o) = S$, $S(i+1) =$ *the subgroup of* G *which is generated by all commutators* $c = [s, g]$, *with* c *and* s *in* $S(i)$, g *in* G.

For π a set of primes, let us denote by $P(\pi)$ the subgroup of G which is generated by all those elements of G whose orders have no prime divisors in π. A central theorem of our discussion is the following: if π contains all the prime divisors of the index $[S:S(i)]$ for some i, then every prime which divides $[P(\pi) \wedge S:P(\pi) \wedge S(i)]$ also divides $[P(\pi):P(\pi) \wedge S]$. From this result we conclude in particular a generalization of Burnside's theorem: if $S(i) = 1$ for some i — in which case we call S *hyperfocal* — and if S has order prime to its index in G, then there exists a normal subgroup N of G such that $G = NS$ and $1 = N \wedge S$. Thus a group is nilpotent if and only if each of its Sylow subgroups is hyperfocal.

The only proof of the theorem of Frobenius in the literature involves group characters. In an attempt to give a purely group theoretic proof, Grün has established a generalization of a special case of this theorem [Grün: Beitrage zur Gruppentheorie. II., Crelle's J., 174(1945), pp 165-169]. Our methods lead to a somewhat sharpened form of the theorem of Grün.

VITA

Donald Gordon Higman was born on September 20, 1928, in Vancouver, Canada. After graduating from Kitsilano High School in 1945, he entered the University of British Columbia and received the degree of Bachelor of Arts, with honors in Mathematics, from that institution in May, 1949. He began his graduate work at the University of Illinois in September, 1949 and received the degree of Master of Arts from that institution in February, 1951. He held a half-time graduate assistantship throughout his residence at the University of Illinois.